# Further Issues in Stage Metrics

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Stein and Heikkinen (2009) are mostly right in their characterization of the lack of metric properties of most stage theories. However, they leave out two very important facts. One is that the Model of Hierarchical Complexity (MHC) is a general theory and part of the normal Mathematical Theory of Measurement (Krantz, Luce, Suppes, & Tversky, 1971; Luce, & Tukey, 1964) applied to the phenomena of difficulty. This means that the nature of hierarchical complexity and its implications for measurement are not included in the Stein and Heikkinen account. Another is that the MHC is not limited to scoring applications, such as its Hierarchical Complexity Scoring System (HCSS; Commons, Rodriguez, Miller, Ross, LoCicero, et al., 2007) or Dawson's Lectical Assessment System (LAS; 2008). The MHC is also used to generate tasks for measurement of any entities' performances of tasks (humans, social organizations, animals, and machines) as well as for other applications (e.g., design of neural nets; see Commons & White, 2006/2009).

Without these facts included, Stein and Heikkinen and their readers miss the importance of the theory and its role in the LAS methods they discuss. As a mathematics-based, behavioral developmental theory of measurement, it is a content-free stage-and-transition step generator. As such, it is a paradigm that accounts for the existence of developmental behavioral patterns and, among other things, the stage theories that describe such patterns. These facts bear directly on—and strengthen—Stein and Heikkinen's arguments and should be considered in discussions of developmental metrics and their properties.

## The Nature of Order of Hierarchical Complexity Scale and Transition Scale

The sequence of orders of hierarchical complexity are built out of a very simple definition.

One task is more hierarchically complex than another task if all of the following are true.

a) It is defined in terms of two or more lower-order task actions. In mathematical terms, this is the same as a set being formed out of elements. This creates the hierarchy.

   A = {a, b} a, b are "lower" than A and compose set A

   A ≠ {A,...} A set cannot contain itself. This means that higher order tasks cannot be reduced to lower order ones. For example, postformal task actions cannot be reduced to formal ones.

b) It organizes lower order task actions. In mathematics' simplest terms, this is a relation on actions. The relations are order relations

   A = (a, b) = {a, {b}} an ordered pair

c) This organization is non-arbitrary. This means that there is a match between the model-designated orders and the real world orders. This can be written as: Not P(a,b), not all permutations are allowed.

By induction, one can build the whole sequence of orders of hierarchical complexity using just this definition and the assumption that there are elements. As a theorem, we proved that there can be only one such sequence of orders, the scale of the order of hierarchical complexity being ordinal. They are just counts of the number of times that a higher order action has organized a next order lower order action. Note that Order 1 consists of simple actions that do not organize action. Ordinal scales have indeterminately sized gaps (Luce, May, 2006 personal communication). The following case illustrates this. Person 1 is taller than person 2, who is taller than person 3, placing them in the order 1, 2, 3 from tallest down. But the first person is 6 feet and 1 inch tall, the second person is 6 feet tall, and the third person is 5 feet tall. The gaps between person 1 and 2 are not equal to the gaps for person 2 and 3 yet their ordinal numbers are 1, 2 and 3. Another theorem shows that all stage theories are subsets of the MHC but the reverse is not true (Commons & Pekker, 2009). See Appendix 1 for how the MHC organizes the orders and stages.

The nature of the order of hierarchical complexity scale and the corresponding stage of performance scale are missing from Stein and Heikkinen's account. The orders are absolute, ordinal, and fractal. The orders have the same construction at each order per definition given above. There is no confusion between steps and orders. Not only are the orders in every domain and content the same, the steps are the same irrespective of order of complexity. The order scale and transition steps scale are both fractal as Ross (2008) points out. The transition steps have a more complex mathematical underpinning to them due to their fractal nature. As a consequence of the fractal nature of orders and transition steps, the next stage performance is always an attractor and not an ideal or teleological end.

The measure of hierarchical complexity of a task item, for example in an assessment application, is analytically represented by its Order of Hierarchical Complexity (OHC). The measure of stage of performance on a task item is empirically represented by the Rasch Scaled Score. The Model of Hierarchical Complexity can be used to generate a sequence of vignettes or problems varying in hierarchical complexity. Sequences are administered to participants. The data are Rasch (1960/1980) analyzed. Rasch performance scores on items are converted into Stage scores (See Appendix 2). Stage scores are regressed against the order of hierarchical complexity of each of the items from a sequence. People are not at a stage, their performances may be. We do find distributions of stage even within a single participant on different items of the same hierarchical complexity within a single task sequence. Items are at an order, item performances are at a stage, persons' performances on a set of items are inferred from the stage of the adjacent items in a Rasch analysis. A Rasch Analysis simultaneously minimizes participant and item error.

To contrast it with traditional stage theories, the MHC is not a psychological or mental view of stage. As one can see from the definition above, there is no mention of content, context, participants, gender, specific tasks, domains, type of organism or machine, or any other specific aspects. It does not depend on culture, or economic status. It is not only independent of particular language but language itself. Therefore, the MHC is not philosophical, teleological, or prescriptive. It is not masculine or feminine because it has no content. It is also not participant dependant or psychometric because it is an analytic mathematical theory of task properties and not person performance. The Hierarchical Complexity Scoring System (HCSS) and Dawson's

Lectical Assessment System also do not need committees of experts to figure out exemplars to define orders as did, for example, Kohlberg's early work or Loevinger's. It is not just a developmental theory but enables mapping of developmental theory onto it because it accounts for the underlying structure of such theory. It is also about more than development. It enables more critical thinking about what comprises development and also more rigorous empirical methods because it separates task (order of hierarchical complexity) from performance (stage).

## Comparing Skill Theory and the Model of Hierarchical Complexity

The closest and most compatible stage theory is Fischer's Skill Theory (1980). There are many similarities and some differences. Both theories were conceived of separately but Fischer's (1980) was published first. Fischer (1981, personal communication) suggested the sentential order but does not include it himself. Fischer also suggested the name for the abstract order. Fischer does not recognize the paradigmatic and cross paradigmatic stages (see Appendix). The similarities include the following. The notion of mapping is probably translatable in the organizing of actions. The levels and orders are roughly the same. There are three differences. Skill theory has tiers, which MHC does not. The tiers seem to limit the existence of the paradigmatic and cross paradigmatic orders and make it hard to add the sentential order. Rasch analysis (Karabatsos, 2001) is also part of Mathematical Theory of Measurement. The mathematical basis of the MHC, and the implications thereof, is what most distinguishes it from Fischer's skill theory.

## History of the Model of Hierarchical Complexity

The Model of Hierarchical Complexity has developed over four periods

1. General Stage Model (GSM) 1 (Commons, & Richards, 1984). 1978-1983. Did not entirely separate task properties and performance properties. Signal detection was used to measure performance. Axioms had serious problems.

2. General Stage Model (GSM) 1984-1998. (Commons, Trudeau, Stein, Richards, & Krause, 1998). Task and performance were separated. Essentially correct informal axioms and definitions introduced. The transition steps were introduced. A scoring manual was developed. At the beginning of the period 1997 to 2006, Dawson had suggest that the gap sizes might be equal. This turned out not to be true as Luce (May, 2006, personal communication) pointed out.

3. Model of Hierarchical Complexity 1997-2008. Dawson (Personal communication, 1997; 2002, 2003) introduced the name Model of Hierarchical Complexity and Rasch analysis for measurement. Commons and Pekker (2008; Commons, Goodheart, Pekker, Dawson, Draney, & Adams, 2008) mathematically formalized the definitions and axioms to more closely meet the Theory of Measurement standards. The ordinal nature of the orders was more clearly articulated along with the indeterminate nature of gaps when they exist. Ross (2008) introduced the fractal nature of orders, stages, and steps.

4. MHC in mathematical refinement stage. The last of the problems of meeting the mathematical criteria in the Theory of Measurement are being addressed. Ross is doing

further research on transition steps and seeking collaborators to investigate how the fractal nature of transitions can be mathematically represented. Precision teaching has been applied to the issue of scoring and item construction.

As one can see from the periods the MHC can have faults, which so far have been able to be overcome. It is a testable theory: if there was an additional order discovered in between the orders now asserted, the theory would need revision (Falmange, 2009, personal communication)

## Using the Model of Hierarchical Complexity

In mapping the mathematical orders of order of hierarchical complexity and of stage transition on to real world data, there are a number of considerations. Because the model does not call for global measures (e.g., of a person's "center of gravity"), it is possible to look at change trial by trial, choice by choice, task action by task action. Performance measures can be applied at any time scale to help uncover process.

The methodology is also flexible in contrast to instrument-dependent stage theories (e.g Loevinger, 1976, 1978; Rest, 1979 Rest, Narvaez, Bebeau, & Thoma, 1999). One can give tasks that are open ended, close ended using choice or preference, or rating scales for example with vignettes. One can even give tasks that vary in Order of Hierarchical Complexity (OHC) to animals who do not talk and also to computers. The mapping is not sample dependent and is not psychometric but analytic at its base. Because the MHC is analytic it can be mathematically true but not useful. Hence it is true that it does not depend on data but it is extremely useful in seeing if applications performed as predicted. For example, the order of hierarchical complexity predicts the Rasch scaled stage score performances on the same items with *r*'s ranging from .85 to .99. By contrast, other stage theories have no such independent variable much less one that works as well as order of hierarchical complexity (see Dawson, 2006).

Because of the flexibility of the time scale for both assessing stage and transition step, it is possible to determine within person distributions of both stage (Fischer, Stein, & Heikkinen, 2009), and step (Ross, 2008). It is possible to get longitudinal data of those distributions. The steps also have their distribution both within a trial or across trials. Theoretically one can make problems that determine the steps at a given stage so that interviewing and scoring are not necessary. The model helps understand the non-stage aspects of development. By examining the residuals after its effects of the Order of Hierarchical Complexity (OHC) it possible to identify other dimensions of difficulty and development as measured by performances.

## Conclusion

Stein and Heikkinen's (2009) restrictive view of the MHC, limiting it to the scoring of speech and writing, is apparently because their main exposure is to Dawson's application of it in the LAS. The restriction does not do justice to the scope and power of the model, which largely enables their discussion of validity, metrics, and measurement in developmental theory. One implication of the model providing the content-free, underlying structure of developmental stages is that other stage theories could be mapped into the orders of hierarchical complexity and thus benefit from the same kind of strength in validity and metrics as the LAS benefits from.

Then, rather than being merely inductive schemes that cluster behavioral characteristics, they could rest on—and draw from—hierarchical complexity's strong theoretical and methodological bases, which currently they do not.

In addition to responding to Stein and Heikkinen's article, this writing would be incomplete without mentioning some final methodological implications of the model. One can have a true comparative cognition and evolutionary behavioral developmental account of the increased power or reasoning, problem solving, emotion, etc. (see Commons, 2006). The determination of power is based on making actual comparisons across different animal species and machines. This in turn leads to the possibility of constructing stacked neural networksⓅ (Commons, 2008; Commons & White, 2006/2009). Stacked neural networks are based on the recapitulation of evolution as shown by the increase in the number of neural network stacks, with potential to have actually intelligent computers and droids.

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# Appendix 1

**Orders of Hierarchical Complexity and Structures of Tasks**

| Order Ordinal and Name | General descriptions of tasks performed |
|---|---|
| 0 Calculatory | Exact without generalization. Example: simple machine arithmetic on 0s, 1s |
| 1 Sensory or motor | Discriminate in a rote fashion, stimuli generalization, move; move limbs, lips, eyes, head; view objects and movement. Discriminative and conditioned stimuli. Example: Either see circles, squares, etc., or instead, touch them. O □ |
| 2 Circular sensory-motor | Form open-ended classes; reach, touch, grab, shake objects, babble; Open ended classes, phonemes. Example: Reach and grasp a circle or square. O □ |
| 3 Sensory -motor | Form concepts; respond to stimuli in a class successfully. Morphemes, concepts. Example: A class of open squares may be formed □ □ □ □ □ |
| 4 Nominal | Find relations among concepts. Use names and use them and other words as successful commands. Single words may be ejaculatory and exclamatory, and include verbs, nouns, numbers= names, letters= names. Example: That class may be named, ASquares.@ |
| 5 Sentential | Imitate and acquire sequences; follow short sequential acts; generalize match-dependent task actions; chain words together. Use pronouns. Example: The numbers, 1, 2, 3, 4, 5 may be said in order. |
| 6 Pre-operational | Make simple deductions; follow lists of sequential acts; tell stories. Count random events and objects; combine numbers and simple propositions. Use connectives: as, when, then, why, before; products of simple operations. Example: The objects in a row of 5 may be counted; last count called 5, five, cinco, etc.<br>* * * * * □ □ □ □ □ O O O O O □ / "} Q |
| 7 Primary | Simple logical deduction and empirical rules involving time sequence. Simple arithmetic. Can add, subtract, multiply, divide, count, prove, do series of tasks on own. Times, places, counts acts, actors, arithmetic outcome from calculation. Example: There are behaviors that act on such classes that we call simple arithmetic operations. 1 + 3 = 4; 5 + 15 = 20; 5(4) = 20; 5(3) = 15 |

| | |
|---|---|
| 8 Concrete | Carry out full arithmetic, form cliques, plan deals. Do long division, follow complex social rules, take and coordinate perspective of other and self. Use variables of interrelations, social events, what happened among others, reasonable deals. Example: There are behaviors that order the simple arithmetic behaviors when multiplying a sum by a number. Such distributive behaviors require the simple arithmetic behavior as a prerequisite, not just a precursor. $5(1 + 3) = 5(1) + 5(3) = 5 + 15 = 20$ |
| 9 Abstract | Discriminate variables such as stereotypes; use logical quantification; form variables out of finite classes based on an abstract feature. Make and quantify propositions; use variable time, place, act, actor, state, type; uses quantifiers (all, none, some); make categorical assertions (e.g., AWe all die.@). Example: All the forms of five in the five rows in the example are equivalent in value, $x = 5$. |
| 10 Formal | Argue using empirical or logical evidence; logic is linear, one-dimensional; use Boolean logic=s connectives (not, and, or, if, if and only if); solve problems with one unknown using algebra, logic, and empiricism; form relationships out of variables; use terms such as if…then, thus, therefore, because; favor correct scientific solutions. Example: The general left hand distributive relation is<br>$x * (y + z) = (x * y) + (x * z)$ |
| 11 Systematic | Construct multivariate systems and matrices, coordinate more than one variable as input; situate events and ideas in a larger context, i.e., considers relationships in contexts; form or conceive systems out of formal relations: legal, societal, corporate, economic, national. Example: The right hand distribution law is not true for numbers but is true for proportions and sets.<br>$x + (y * z) = (x * y) + (x * z)$; $x \cup (y \cap z) = (x \cap y) \cup (x \cap z)$<br>*Symbols:* $\cup$ = union (total elements); $\cap$ = intersection (elements in common) |
| 12 Meta-systematic | Integrate systems to construct metasystems out of disparate systems; compare systems and perspectives in a systematic way (across multiple domains); reflect on and name properties of systems. Example: The system of propositional logic and elementary set theory are isomorphic.<br>x & (y or z) = (x & y) or (x & z) Logic; $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$ Sets<br>T(False) $\Leftrightarrow$ $\varphi$ Empty set; T(True) $\Leftrightarrow$ $\Omega$ Universal set<br>*Symbols:* & = and; $\Leftrightarrow$ = is equivalent to; T = Transformation of |
| 13 Paradigmatic | Integrate or discriminate how to fit metasystems together to form new paradigms. Includes ability to show that there are no ways to fit together any set of metasystems. $\Omega_1 \circ \Omega_2 = \Psi^a$<br>*Symbols:* $\Omega_n$ = e.g., Algebraic Metasystems; $\Omega_n$ = e.g., Geometric Metasystems; $\Psi^a$ = Analytic Geometry as a paradigm |
| 14 Cross-paradigmatic | Integrate, compare, reflect on and name properties of paradigms within or across domains. Fit paradigms together to form new fields. |

*Note:* From "Applying the Model of Hierarchical Complexity" (p. 65), by Commons, Rodriguez, Miller, Ross, LoCicero, Goodheart, & Danaher-Gilpin. 2007. Cambridge, MA: Dare Association, Inc. Copyright 1991-2007 by Dare Association, Inc. Adapted and reprinted with permission.

# Appendix 2

Because of the discussion of metrics in Stein and Heikkinen (2009), it is important to point out the difference in formal measurement theory terms between the orders of hierarchical complexity, which is an ordinal scale analytic measure, and the corresponding measure of performance Rasch scaled item and person scores which is conjoint. As Perline, Wright, and Wainer, (1979) state, “the Rasch (1960) model is a practical realization of conjoint measurement (Brogden, 1977; Fischer, 1968; Keats, 1967, 1971). Tversky (1967) discussed the Bradley-Terry-Luce choice model, which is closely related to the Rasch model, in terms of conjoint measurement. Young (1972) also considered the Bradley-Terry-Luce model in these terms and remarked generally that "the scaling methods in psychometrics conform to the notion of polynomial conjoint measurement.”

One would like to have a person stage of performance score. But there is no simple and direct way of obtaining that score from a Rasch analysis. A Rasch scaling procedure minimizes errors of fit to both persons (participants) and items at the same time. The Rasch map displays the person performance scaled values on the left side of a linear vertical scale and the items scores on the right side of that same scale. But the problem is that Rasch scores are not in the stage metric. And, to make things more curious, the order of hierarchical complexity is ordinal and cannot be averaged, summed, or even subtracted. But the Rasch scores can. Hence the interpolation between adjacent orders of hierarchical complexity of items is based on the Rasch scale for which interpolation is fine. This is because the Rasch scale is a conjoint measure which allow for summing.

To find the person performance stage from the adjacent corresponding items’ order of hierarchical complexity, one can interpolate between the items’ hierarchical complexity. There is no assumptions about the size and nature of a possible gap. The Rasch scale is linear between OHC because Rasch is a conjoint measure. This can be done by translating the Rasch scores into stage of performance scores based on the corresponding absolute values of the order of hierarchical complexity of the items. They represent the stage of a person’s performance according to the Model of Hierarchical Complexity. A person’s stage of performance score is calculated by using the equation:

$$StageofPer\,son = \frac{PersonRasc\,hScore - StageMean_1}{StageMean_2 - StageMean_1} + ItemHC$$

Stage of Person = Stage of performance of person in Order of Hierarchical Complexity number.

Person Rasch Score = The Rasch Scaled performance score for a person.

Stage Mean = Mean Rasch Score for items of a given hierarchical complexity.

Item HC = Items order of hierarchical complexity.

This relationship between the two scales allows for using local interpolating between items with adjacent orders and not overall. After this transformation the obtained stage of performance scores for persons can be compared. This is useful in doing a factor analysis.